Absolute Instability in Hot Jets

Peter A. Monkewitz*
University of California, Los Angeles, Los Angeles, California
and
Kiho D. Sohn†
Hughes Aircraft Company, Los Angeles, California

The linear inviscid stability of heated compressible axisymmetric jets is re-examined, focusing on the impulse response of the flow. The case is of particular interest in which a transient grows exponentially in place, i.e., at the location at which it was generated. Such a situation is commonly termed absolutely unstable as opposed to convectively unstable. Limiting the analysis to locally parallel mean velocity and density profiles, infinite Froude number, and zero Eckert number, the absolute instability boundaries, i.e., the boundaries between absolute and convective instability, are explored in the mean flow parameter space. The influence of mean flow profiles, exit Mach number, exit temperature ratio, and exit velocity ratio are considered and discussed separately. It is shown that heated jets, i.e, jets with lower than ambient density, can, depending on operating conditions, develop a region of local absolute instability in the potential core region if the jet density is less than 0.72 times the ambient density. The results suggest that a self-excited global resonance may dominate the heated jet response in situations of practical interest.

I. Introduction

IT has been suggested by several authors, among them Crow,[1] Moore,[2] Ffowcs-Williams and Kempton,[3] Tam and Morris,[4] and Huerre and Crighton,[5] that even in the turbulent regime of practical interest linear stability theory, together with assumptions about the evolution of the mean flow or directly about the evolution of the instability wave amplitude, is able to model the growth and decay of "large-scale structures" thought to be mainly responsible for the low-frequency end of the radiated noise. From this viewpoint, the linear stability analysis of compressible heated jets is of particular interest, as it relates to jet engine exhaust noise. It has been studied, notably by Michalke[6] and Morris,[7] who considered spatially developing disturbances, i.e., the response of the jet to a localized steady-periodic forcing. The problem of its impulse response, on the other hand, which is the main subject of this study, has not been addressed until very recently, when it was briefly discussed by Huerre and Monkewitz[8] (Sec. V) and more completely by Sohn.[9]

In Sec. II of this paper, the mean flow for the stability analysis is specified to match available experimental data. In particular, a new two-parameter family of mean velocity and related density profiles is introduced that allows for both variations in the velocity ratio R (the difference of the jet centerline and exterior uniform velocity divided by its sum) and for smooth variations in the profile from a discontinuous "tophat" to a Gaussian. In accord with the observation by Smith and Johannesen[10] that buoyancy plays a secondary role in the problem of noise radiated from hot jets with a velocity of practical interest, only the limit of infinite Froude number is considered. In addition, the Eckert number is assumed to be zero; i.e., when relating the mean temperature or density to the velocity profile, the generation of heat by viscous dissipation is neglected.

The stability analysis of compressible heated jets, which closely follows Michalke,[6] is developed in Sec. III. In Sec. IV, the linear impulse response is examined in more detail. Thereby, two different responses are distinguished:[8,11-13] in one case, termed *absolutely unstable*, a locally generated small-amplitude transient grows exponentially at the location of its generation and eventually contaminates the entire (parallel) flow region. In the other case, called *convectively unstable,* the transient or wavepacket is convected downstream and leaves the mean flow ultimately undisturbed. This distinction is briefly reviewed, and the application of the Briggs[11]-Bers[12] criterion for absolute instability is discussed. A short mention is also made of the bearing of these concepts on real nonparallel flows.

The remainder of the paper is devoted to the exploration of local absolute instability in the heated compressible jet. The main results are presented as absolute instability boundaries in the mean flow parameter space. Both the axisymmetric and the first spinning mode are investigated, and the effect of the temperature ratio, Mach number, velocity ratio, and profile shape is considered.

II. Modeling of the Jet Mean Flow

In order to perform the stability analysis, realistic mean velocity and density profiles have to be specified. As long as the flow is treated as locally parallel, these profiles can be picked arbitrarily at each station. If results at different x stations are to be compared, however, the sections have to be related in a self-consistent fashion. The present approach outlined next is somewhat similar to the one taken by Tanna and Morris.[14]

First, a two-parameter family of velocity profiles $u(r)$, defined by Eq. (1), has been developed to describe, for arbitrary velocity ratios R, a profile anywhere between a cylindrical vortex sheet ($N=\infty$) and a Gaussian ($N=1$), which is shown in Refs. 15 and 16 to be the most appropriate for the asymptotic region. Throughout the paper, velocities are made nondimensional with the average mean velocity $\bar{u}^* = [u_c^* - u_\infty^*]/2$, where an asterisk denotes a dimensional quantity and the subscripts c and ∞ refer, respectively, to the jet centerline and the (ambient) region far away from the jet. The length scale implied by Eq. (1) is the local jet radius r_j^*, defined by $u^*(r_j^*) = \bar{u}^*$:

Presented in different form as Paper 86-1882 at the AIAA 10th Aeroacoustics Conference, Seattle, WA, July 9-11, 1986; received Aug. 14, 1986; revision received Dec. 14, 1987.

*Associate Professor, Department of Mechanical, Aerospace and Nuclear Engineering. Member AIAA.

†Engineer, Radar Systems Group; currently Member, Technical Staff, Rocketdyne Corporation.

$$u(t)=1-R+2RF(r) \quad (1)$$

$$F(r)=\{1+[\exp(r^2\ln 2)-1]^N\}^{-1}$$

$$N^{-1}=\begin{cases}0.02+0.869(x/L)-0.031(x/L)^{6.072}, & x<1.35L\\ 1, & x\geq 1.35L\end{cases}$$

$$L=[8.0+2.5\ln(S_e)]/R$$

$$\delta_\omega \sim (N\ln 2)^{-1},\ N\rightarrow\infty \quad (2)$$

The normalized profile $F(r)$ is shown in Fig. 1 for different values of the parameter N, which controls the jet mixing layer thickness. Hereafter, N is prescribed as a function of x/L, where the potential core length L depends on the exit density ratio $S_e=\rho_{c,e}/\rho_\infty$ and the velocity ratio R. The relationship has been chosen to yield a vorticity or maximum slope thickness δ_ω, given asymptotically by Eq. (2), which initially grows linearly with downstream distance x in accord with the measurements of Brown and Roshko.[17] The initial $N=50$ at $x=0$ is chosen arbitrarily to represent a typical laboratory jet.

The next step is to specify the relationship between the velocity and density profiles. The concept of turbulent Prandtl number is used in the form proposed by Reichardt and discussed in Schlichting,[15] Chap. XXIV. In the present notation, it is expressed by

$$\frac{1/\rho-1}{1/S-1}=\left(\frac{u+R-1}{2R}\right)^P \quad (3)$$

$$P=\begin{cases}1 & x\leq 0.64L\\ 0.8+0.2\cos[2.49(x/L)-1.60] & 0.64L<x<1.90L\\ 0.6 & 1.90L\leq x\end{cases}$$

The Prandtl number P is again specified as a function of x so that the result agrees with the measurements of Brown and Roshko[17] for small x and Corrsin and Uberoi[16] in the asymptotic region. Finally, the development of the dimensional centerline velocity is selected so that it matches the data of Corrsin and Uberoi.[16] The expression

$$\frac{u_c^*-u_\infty^*}{u_{c,e}^*-u_\infty^*}=1.682[(x/L)^7+37.1(x/L)^{-0.189}]^{-1/7} \quad (4)$$

valid for $x\geq L$, has been found most suitable as it displays a decay inversely proportional to x in accord with the asymptotic scaling laws for turbulent jets. If the velocity ratio R_e, the density ratio S_e, and the Mach number M_e at the jet exit are specified, the local velocity ratio R and Mach number M are directly obtained from Eq. (4). To obtain the local density ratio S and the ratio of local to exit jet diameter $(2r_j^*)/D^*$, the conservation of momentum and energy is used, neglecting buoyancy and heat generated by viscous dissipation and assuming a uniform ratio of specific heats (for details see Ref. 9). The task of "creating" a self-consistent jet mean flow is then complete. A typical result of the outlined procedure is shown in Fig. 2, where only the velocity is plotted, since the temperature has qualitatively the same appearance.

III. Linear Stability Analysis

In this section, our stability analysis follows Michalke[6,18] quite closely, except for the nondimensionalization, which is specified in the previous section. To arrive at the following formulation several simplifying assumptions are made. First, nonlinear terms are neglected, which limits the analysis to the small-amplitude regime. Second, the mean flow is assumed to be locally parallel. In other words, it is assumed that the axial mean flow variation is slow on the scale of the instability wave. No corrections for the flow divergence as in the "slowly diverging" approach are considered in this paper. Third, the mean flow, which in many applications is turbulent, is treated

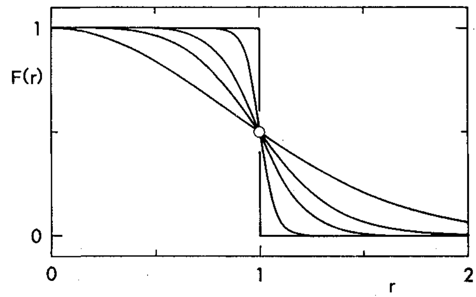

Fig. 1 The normalized velocity profile $F(r)$, defined by Eq. (1), for $N^{-1}=1$, 0.5, 0.3, 0.1, and 0.

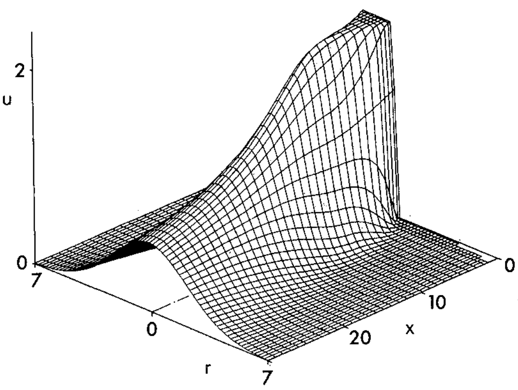

Fig. 2 Example of the jet velocity vs r and x for $R_e=1$, $S_e=0.5$, and $M_e=0$.

as "quasilaminar." That is, the interaction of the large instability scales with small-scale turbulence is neglected, as discussed explicitly by Strange and Crighton.[19] Fourth and last, buoyancy is neglected, and only the limit of infinite Reynolds number is considered. The rationale is that, given all the other simplifications, the relatively minor quantitative differences between inviscid and viscous calculations (see, e.g., Morris[7]) do not justify the considerable increase of computation associated with finite Reynolds numbers. Since the mean flow defined in Sec. II is axisymmetric, it is useful to represent every disturbance quantity q'' as a Fourier series in θ, given by

$$q''(t,x,r,\theta)=\sum_{m=0}^{\infty}q'(t,x,r;m)\exp(im\theta) \quad (5)$$

Using this representation and assuming that the medium is an ideal gas with uniform composition, the linearized equations for the streamwise, radial, and azimuthal disturbance velocity components u',v', and w' and the disturbance pressure and density p' and ρ' are obtained in the form of Eq. (6). The ambient mean density ρ_∞^* has thereby been chosen as reference, and the local Mach number M is based on centerline velocity and ambient sound speed. Note that, in terms of the velocity ratio R, the nondimensional centerline velocity becomes $u_c=1+R$, while the ambient mean velocity is $u_\infty=1-R$:

$$\frac{D\rho'}{Dt}+\frac{d\rho}{dr}v'+\rho\left[\frac{\partial u'}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(rv')+\frac{im}{r}w'\right]=0 \quad (6a)$$

$$\frac{Du'}{Dt}+\frac{du}{dr}v'=-\frac{1}{\rho}\frac{\partial p'}{\partial x} \quad (6b)$$

$$\frac{Dv'}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial r} \tag{6c}$$

$$\frac{Dw'}{Dt} = -\frac{im}{r\rho}p' \tag{6d}$$

$$\rho\left(\frac{M}{1+R}\right)^2\frac{Dp'}{Dt} - \frac{D\rho'}{Dt} - \frac{d\rho}{dr}v' = 0 \tag{6e}$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} \tag{6f}$$

The solution of the system of Eqs. (6) is sought in the usual fashion in terms of normal modes

$$q'(t,x,r) = \tilde{q}(r)\exp(ikx - i\omega t) \tag{7}$$

where the azimuthal mode number m has been dropped in the argument. The result is the following set of two coupled ordinary differential equations for $\tilde{p}$ and $\tilde{v}$ which, together with the boundary conditions of Eq. (8c), represents an eigenvalue problem with either the wave number k or the complex phase speeed $c = \omega/k$ playing the role of eigenvalue:

$$ik\rho(u-c)\tilde{v} = -\frac{d\tilde{p}}{dr} \tag{8a}$$

$$ik\rho\left[(u-c)\frac{1}{r}\frac{d}{dr}(r\tilde{v}) - \frac{du}{dr}\tilde{v}\right] = -\left[\lambda^2 + \left(\frac{m}{r}\right)^2\right]\tilde{p} \tag{8b}$$

$$\lambda^2 \equiv k^2\left[1 - \rho\left(\frac{M}{1+R}\right)^2(u-c)^2\right]$$

$$\tilde{p},\tilde{v} < \infty \quad \text{at } r=0$$

$$\tilde{p},\tilde{v} \to 0 \quad \text{as } r\to\infty \tag{8c}$$

Following Michalke,[6] the problem may be further simplified to one nonlinear ODE by setting

$$\chi = -\frac{ik\tilde{p}}{\tilde{v}} \tag{9}$$

with the final result

$$\frac{d\chi}{dr} = -k^2\rho(u-c) + \chi\left\{\frac{1}{u-c}\left[\frac{\lambda^2 + (m/r)^2}{k^2\rho}\chi - \frac{du}{dr}\right] + \frac{1}{r}\right\} \tag{10a}$$

$$\chi \to \chi_c = -\frac{k^2 S(1+R-c)}{\lambda_c}\frac{I_m(\lambda_c r)}{I'_m(\lambda_c r)} \quad \text{as } r\to 0 \tag{10b}$$

$$\lambda_c^2 = k^2\left[1 - S\left(\frac{M}{1+R}\right)^2(1+R-c)^2\right]$$

$$\chi \to \chi_\infty = -\frac{k^2(1-R-c)}{\lambda_\infty}\frac{K_m(\lambda_\infty r)}{K'_m(\lambda_\infty r)} \quad \text{as } r\to\infty$$

$$\lambda_\infty^2 = k^2\left[1 - \left(\frac{M}{1+R}\right)^2(1-R-c)^2\right] \tag{10c}$$

Since the radial gradients du/dr and $d\rho/dr$ vanish on the centerline and far from the jet, the boundary conditions of Eq. (8c) easily lead to the two limiting solutions Eqs. (10b) and (10c) in terms of the modified Bessel functions I and K. In these formulas, the prime denotes a derivative with respect to the argument, and S is the local ratio of centerline to ambient density. The numerical integration of Eq. (10a) is now straightforward if the logarithmic singularity of the limiting solution Eq. (10b) for $m=0$ is properly handled (for details see Ref. 9). If the velocity and density profiles are discontinuous with a cylindrical vortex sheet separating the uniform jet from the ambient flow, the numerical integration of Eq. (10a) is replaced by the application of suitable jump conditions across the discontinuity, i.e., the continuity of pressure and displacement.

IV. Absolute and Convective Instability

In the following, the response of the jet flow to a "ring pulse" with strength proportional to $\exp(im\theta)$ at, say, time $t=0$, a downstream location $x=x_0$, and a radius $r=r_0$ is considered. The analysis in this case is completely analogous to the one in the two-dimensional case given, for instance, by Huerre and Monkewitz,[8] and the reader is referred to Refs. 8, 11–13, and 20 for details. The result of these analyses, which all involve a double Fourier transform in t and x, may be summarized as follows. Using asymptotic methods to evaluate the large-time behavior of the impulse response, they show that along each ray $x/t=$ const, the mode with wave number k^s, defined by Eq. (11a), dominates over all other discrete modes and evolves in time as $t^{-1/2}\exp(-i\sigma t)$ [see, e.g., Eq. (13) in Ref. 8], where σ is given by Eq. (11b):

$$\frac{d\omega}{dk}(k^s) = \frac{x}{t} \tag{11a}$$

$$\sigma = \omega^s - k^s x/t, \qquad \omega^s = \omega(k^s) \tag{11b}$$

Technically speaking, Eq. (11a) arises from a point of stationary phase $d(kx-\omega t)/dk = 0$ in the k plane, which provides the leading contribution to the inverse Fourier transform in k, also referred to as the "wave packet integral." The temporal growth rate along each ray $x/t=$ const is now simply given by

$$\sigma_i = \omega_i^s - k_i^s\frac{d\omega}{dk}(k^s) \tag{12}$$

In general, the wave packet is confined within a wedge bounded by two rays of zero amplification rate $\sigma_i = 0$, as shown in the (x,t) plane of Fig. 3. Outside this wedge, disturbances are decaying algebraically, while within the wedge, they grow exponentially. The ray along which the wave packet experiences the largest growth is thereby easily shown to correspond to the wave number k^s of maximum temporal growth (see, e.g., in Ref. 21). As depicted in Figs. 3a and 3b, the impulse response can behave in two qualitatively different ways. If a flow is unstable and the edges of the wave packet travel in opposite directions with the ray $x/t=0$ included in the unstable wedge (Fig. 3a), the flow is called *absolutely unstable*. If, on the other hand, the edges of the wave packet travel in the same direction (Fig. 3b), the flow is *convectively unstable*. The relevant frame of reference, as noted by Lifshitz and Pitaevskii,[13] is thereby uniquely defined in most practical problems, especially where solid boundaries such as a jet tailpipe are present. The distinguishing feature between the two cases of Fig. 3 is the growth rate of the wave packet along the ray $x/t=0$ (the vertical axis on Fig. 3). From Eq. (11a), it follows that the wave number associated with the ray $x/t=0$

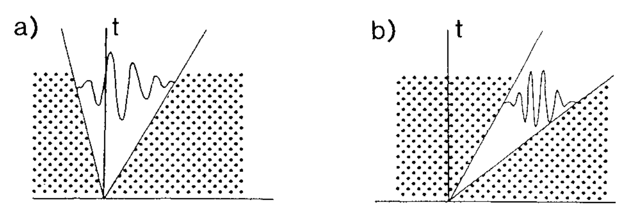

Fig. 3 Sketch of a typical impulse response: a) absolutely unstable flow; b) convectively unstable flow (shaded areas are regions with $\sigma_i < 0$).

and denoted by k^0 belongs to a mode with zero group velocity $(d\omega/dk)(k^0)=0$. Its growth rate, henceforth called *absolute growth rate*, is simply given by $\sigma_i=\omega_i^0$, with $\omega^0=\omega(k^0)$. Since the frequency ω^0 is in general a branch point of order two, i.e., locally a square root branch point of the dispersion relation $k(\omega)$, the following criterion attributed to Briggs[11] and Bers[12] can be formulated. For an unstable flow to be absolutely unstable, two conditions must be satisfied. First, at least one of the branch point singularities of its dispersion relation must lie in the upper half of the ω plane (i.e., some $\omega_i^0>0$). Second, one has to ascertain that a branch point with $\omega_i^0>0$ arises from the coalescence of a downstream mode $k^+(\omega)$ and an upstream mode $k^-(\omega)$. Otherwise the flow is convectively unstable. The second condition means in technical terms that the maps $k^+(\omega)$ and $k^-(\omega)$ of contours parallel to the real ω axis must separate into the upper and lower halves of the complex k plane, respectively, when these contours are placed high enough above the branch point ω^0 (see Ref. 8, Fig. 2, in particular). This condition is the so-called pinching requirement[11,12] for the integration contour of the wave packet integral in the k plane. A verification of this requirement in one representative case is shown as Fig. 7 in Sec. V.

At this point, the reader is reminded that the previously discussed concepts pertain to purely parallel flows. Their relationship to real nonparallel flows such as the heated jet under consideration is a topic of current reseach. Accounts of it may be found in Refs. 21-26. These studies suggest, in short, that the presence of local absolute instability in a flow may, in certain cases, lead to global self-excited resonances of the system. The related issue of the susceptibility of a system to external disturbances (including control inputs) is exposed in Monkewitz and Sohn[20] and Strykowski.[27]

To arrive at the results in the following sections, Eq. (10a), subject to the boundary conditions of Eqs. (10b) and (10c), was integrated using a standard fourth-fifth-order Runge-Kutta-Fehlberg scheme with automatic step-size control. The dispersion relation $k(\omega)$ or $\omega(k)$ for complex k and ω was determined by shooting from the jet centerline and from large r and by matching the solutions at $r=1$.

Computationally, the character of an instability was determined by tracking branch point singularities of the dispersion relation $k(\omega)$, i.e., by applying the Briggs-Bers critertion. For this, it is not necessary to obtain complete maps of $k(\omega)$ around ω^0 or equivalently $\omega(k)$ around the saddle point k^0, except for the verification of the pinching requirement. Instead, the branch point ω^0 is determined directly by the following procedure: for two suitably selected initial frequencies $\omega^{(1,2)}$ the eigenvalues $k^+[\omega^{(1,2)}]$ and $k^-[\omega^{(1,2)}]$ on both Riemann sheets emanating from the branch point ω^0 are computed, and the expression

$$k^{\pm}-k^0=\pm s(\omega-\omega^0)^{1/2}+l(\omega-\omega^0) \tag{13}$$

is fitted to these four eigenvalues. That is, the constants s and l as well as k^0 and ω^0 are determined. Then, two new frequencies $\omega^{(3,4)}$ closer to the extrapolated branch point ω^0 are chosen automatically, and the procedure is repeated until both ω^0 and k^0 become stationary (to at least three significant digits).

V. Survey of Absolute Instability Boundaries in Compressible Heated Jets

In this section, the stability characteristics of compressible heated jets as modeled by the mean flow of Sec. II is explored. For clarity, the influence of each mean flow parameter is considered separately. Also, to investigate the influence of Mach number and velocity ratio, only the cylindrical vortex sheet with $N=\infty$ is considered for simplicity.

Effect of Mach Number and Asimuthal Wave Number

These effects are studied for the cylindrical vortex sheet only, with velocity ratio fixed at $R=1$ in order to reduce the number of parameter variations. As already reported in Ref. 8, the transition density ratio for the axisymmetric mode is $S_t(N=\infty, m=0)=0.66$ at a Mach number of zero. In completing this survey, it turns out that increasing Mach number has a stabilizing effect, i.e., decreases S_t, or, in other words, requires a hotter jet for absolute instability. This is shown in Fig. 4, which was generated as follows. To obtain one point on the transition curve $S_t(M)$, the branch point ω^0 was calculated for a fixed M and different S until the transition ratio S_t with $\omega_i^0=0$ was found. The trend with Mach number, which applies to the vortical instability mode, seems to agree with the experimental findings of Papamoschou and Roshko[28] and standard stability calculations. Surprisingly, S_t becomes very small already for moderate subsonic Mach numbers, and at $M=0.67$ the real part of the wavenumber k_r^0 becomes zero, a phenomenon with no obvious physical interpretation.

Also included in Fig. 4 are results for the first helical mode $m=1$ where, again, the highest transition density ratio is found at $M=0$ and has the value $S_t(N=\infty, m=1)=0.35$, which is considerably lower than for the axisymmetric mode. The parameter listed along the transition curves in Fig. 4 is the branch point Strouhal number, which is simply $St=\omega^0/2\pi$ (ω^0 being real on the transition curves).

Effect of Mean Velocity and Density Profiles

To test the sensitivity of the stability calculations to the particular functional form of the velocity and density profiles, a comparison with the spatial stability calculations of Michalke[6] was made on the basis of equal vorticity thickness. Despite the differences in profiles, the argreement was found to be excellent in all cases considered. As an illustration, the comparison for the axisymmetric mode $m=0$ at Mach number $M=0$ and density ratio $S=1$ is shown in Fig. 5. It is only reported here that for $S=0.5$, Michalke's "mode II" results have also been faithfully reproduced. As discussed in Ref. 8,

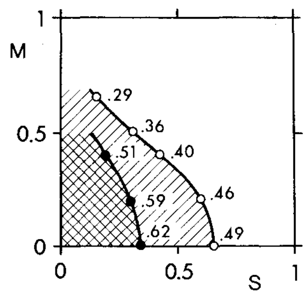

Fig. 4 Absolute instability boundaries for the $m=0$ mode (○) and the $m=1$ mode (●), with $N=\infty$ and $R=1$. The hatched areas are the absolutely unstable regions. The parameter along the curves is the branch point Strouhal number.

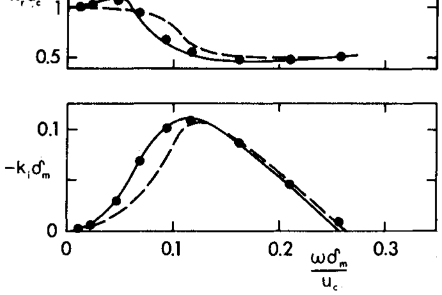

Fig. 5 Comparison with Michalke's spatial calculation[6] for his profile 1, with $m=0$, $M=0$, and $S=1$ and different ratios of the jet radius r_j and shear layer momentum thickness δ_m: —, Michalke's data for $r_j/\delta_m=25$; – – –, Michalke's data for $r_j/\delta_m=12.5$; ●, present data for $r_j/\delta_m=22$.

these mode II branches are directly related to the existence of the branch point ω^0, the location of which determines the absolute or convective nature of the instability. It is, therefore, concluded that the following results are not critically dependent on fine details of the velocity and density profiles.

Next, the effect of finite mixing layer thickness on the absolute instability boundary is investigated. In other words, absolute instability for the jet flow synthesized in Sec. II is investigated as a function of downstream distance x. To find the highest possible density ratio S_e at the jet exit, which leads to absolute instability somewhere in the flow, Fig. 4 suggests that attention be focused on the axisymmetric mode with $M=0$ and $R=1$ (excluding ambient counterflow). The results are shown in Fig. 6, in which ω^0 and k^0 are displayed for a

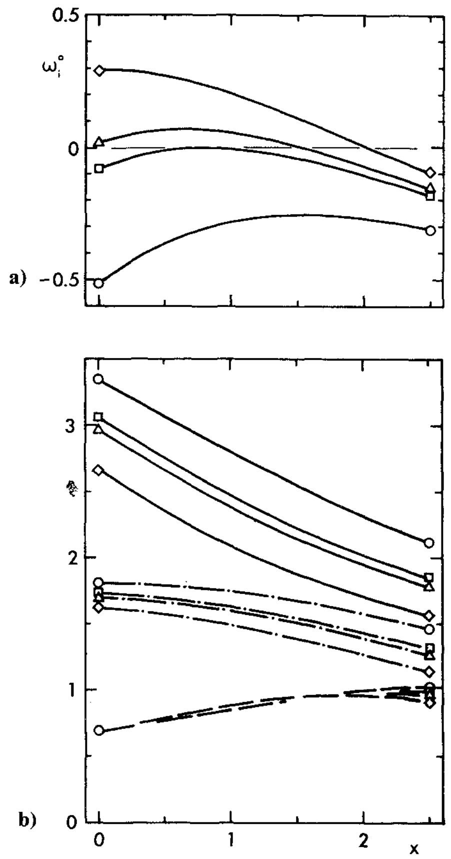

Fig. 6 a) absolute growth rates vs x for the $m=0$ mode and $R=1$, $M=0$, and $S_e=1.0$ (○), $S_e=0.72$ (□), $S_e=0.66$ (△), and $S_e=0.5$ (◇). b) —, corresponding real parts ω_r^0; – – –, corresponding k_r^0; – · – · –, corresponding $-k_i^0$.

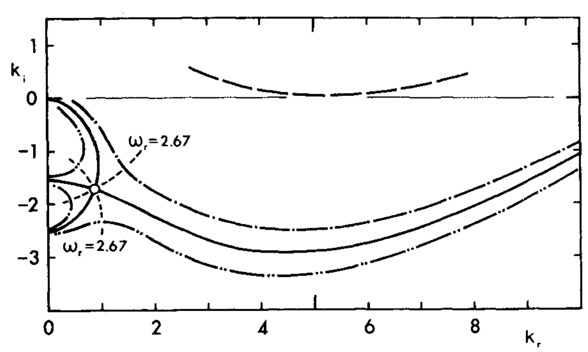

Fig. 7 Maps $k(\omega)$ of contours parallel to the real ω axis for profile [Eq. (1)] with $N=8.56$, $R=1$, $M=0$, and the $m=0$ mode: – – –, contour with $\omega_i^0=2.25$; – · – · –, $\omega_i^0=2.25$; —, $\omega_i^0=0$; – · · · –, $\omega_i^0=-0.05$; -------, contours $\omega_r^0=$ const; ○, saddle point k^0.

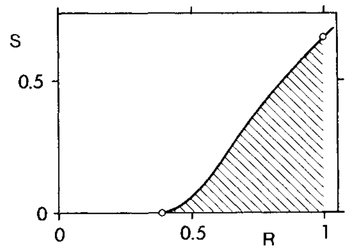

Fig. 8 Absolute instability boundary in the (S,R) plane for $N=\infty$ and $M=0$. The hatched area is absolutely unstable.

cold jet ($S=1$) and several exit density ratios $S_e<1$. The graph of absolute growth rates ω_i^0 shows that marginal absolute instability is reached at a downsteam station (normalized with the jet exit radius) of approximately $x=0.8$ for a transition density ratio $S_{e,t}=0.72$. This point corresponds to a profiles shape parameter $N=8.56$, with a vorticity thickness referred to the jet exit diameter of $\delta_\omega/D=0.087$. Hence, the local absolute instability (the region with $\omega_i^0>0$) first develops away from the nozzle and, as seen in Fig. 5, reaches it only at the density ratio $S_e=0.66$, in perfect agreement with the earlier result[8] for the cylindrical vortex sheet (the exit profile with $N=50$ being almost a vortex sheet).

A representative verification of the pinching requirement is shown in Fig. 7. For this map of $\omega(k)$ around the saddle point or pinching point, the mean flow parameters corresponding to the first marginal absolute instability were chosen. It is evident that in this case the condition is satisfied as the branches $k^+(\omega)$ and $k^-(\omega)$ completely separate into the upper and lower halves of the k plane when $\omega_i>2.25$.

Effect of Velocity Ratio

The effect of a coflowing stream, or the flight effect, is again investigated for the cylindrical vortex sheet only, which has been found to yield good qualitative insight into the mean flow parameter dependence of the absolute instability region. This part of the study is furthermore restricted to zero Mach number. It has already been found by Monkewitz and Huerre[29] that the maximum spatial growth rate of a plane mixing layer is approximately proportional to the velocity ratio R. That is, it decreases as the velocity of the coflowing stream is increased. Very similar behavior has been found for the absolute growth rate ω_i^0. It is seen that the transition density ratio S_t, like the maximum spatial growth rate, is to a good approximation a linear function of R, as shown in Fig. 8. As in Fig. 4, the transition curve $S_t(R)$, or conversely, the transition velocity ratio $R_t(S)$, is again the locus of $\omega_i^0=0$. The region $R>1$, corresponding to ambient counterflow, which is of limited practical interest, has not been investigated. It is noted, however, that the transition point $R_t=1.315$, found in Ref. 8 for the two-dimensional mixing layer with $S=1$, does lie approximately on the linear extrapolation of the transition curve in Fig. 8. At the other end, S_t becomes zero at $R=0.39$ which means that below $R=0.39$ no local absolute instability is possible on a cylindrical vortex sheet at any density ratio (and $M=0$).

VI. Conclusions

In summary, this study has shown that the heated jet can become absolutely unstable on a locally parallel basis for exit density ratios $S_e<0.72$. Furthermore, it has been found that the transition density ratio is lowered when the velocity ratio is decreased, the Mach number increased, and the azimuthal mode number increased. However, additional parameters such as jet turbulence level and buoyancy have been ignored, and the transonic and supersonic regime, in which Ahuja et al.[30] discovered some unexpected behavior of a hot jet when subjected to artificial excitation, has not been explored. Nevertheless, the possibility has been raised that a self-excited global resonance manifests itself in a heated jet under some operating conditions.

Acknowledgments

The support of K.D. Sohn by the Hughes Aircraft Corporation during this work is gratefully acknowleged. The first author also wishes to acknowledge the stimulating discussions with Prof. Alfons Michalke and Dr. Dietrich Bechert, as well as the helpful suggestions of Dr. Mordechai Belöli during the final writeup.

References

[1]Crow, S.C., "Acoustic Gain of a Turbulent Jet," Paper IE6, Annual Meeting of the Division of Fluid Dynamics of the American Physics Society, Boulder, CO, Nov. 20–22,1972.

[2]Moore, C.J., "The Role of Shear-Layer Instability Waves in Jet Exhaust Noise," *Journal of Fluid Mechanics,* Vol. 80, Pt. 2, April 1977, pp. 321–367.

[3]Ffowcs-Williams, J.E. and Kempton, A.J., "The Noise from the Large-Scale Structure of a Jet," *Journal of Fluid Mechanics,* Vol. 84, Pt. 4, Feb. 1978, pp. 673–694.

[4]Tam, C.K.W. and Morris, P.J., "The Radiation of Sound by the Instability Waves of a Compressible Plane Turbulent Jet," *Journal of Fluid Mechanics,* Vol. 98, Pt. 2, May 1980, pp. 349–381.

[5]Huerre, P. and Crighton, D.G., "Sound Generation by Instability Waves in a Low Mach Number Jet," AIAA Paper 83-0661, April 1983.

[6]Michalke, A., "Instabilität eines kompressiblen runden Freistrahls unter Berücksichtigung des Einflusses der Strahlgrenzschichtdicke," *Zeitschrift für flugwissenschaften,* Vol. 19, Aug. 1971, pp. 319–328.

[7]Morris, P.J., "Viscous Stability of Compressible Axisymmetric Jets," *AIAA Journal,* Vol. 21, April 1983, p. 481–482.

[8]Huerre, P. and Monkewitz, P.A., "Absolute and Convective Instabilities in Free Shear Layers," *Journal of Fluid Mechanics,* Vol. 159, Oct. 1985, pp. 151–168.

[9]Sohn, K.D., "Absolute Instability in Hot Jets," M.S. Thesis, University of California at Los Angeles, 1986.

[10]Smith, D.J. and Johannesen, N.H., "The Effects of Density on Subsonic Jet Noise," *Proceedings of the IUTAM Symposium on Aero- and Hydroacoustics,* Springer-Verlag, Berlin, 1986, pp. 445–452.

[11]Briggs, R.J., "Electron-Stream Interaction with Plasmas," *Research Monograph 29,* M.I.T. Press, Cambridge MA, 1964.

[12]Bers, A., "Linear Waves and Instabilities," *Physique des Plasmas,* edited by C. DeWitt and J. Peyraud, Gordon and Breach, New York, 1975, pp. 117–213.

[13]Lifshitz, E.M. and Pitaevskii, L.P., *Physical Kinetics,* Pergamon, Elmsford, NY, 1981.

[14]Tanna, H.K. and Morris, P.J., "The Noise from Normal-Velocity-Profile Coannular Jets," *Journal of Sound and Vibration,* Vol. 98, Pt. 2, Jan. 1985, pp. 213–234.

[15]Schlichting, H., *Boundary Layer Theory,* 6th ed., McGraw-Hill, New York, 1968, pp 703–705.

[16]Corrsin, S. and Uberoi, M.S., "Experiments on the Flow and Heat Transfer in a Heated Turbulent Air Jet," NACA TN 1865, 1947.

[17]Brown, G.L. and Roshko, A., "On Density Effects and Large Structure in Turbulent Mixing Layers," *Journal of Fluid Mechanics,* Vol. 64, Pt. 4, July 1974, pp. 775–816.

[18]Michalke, A., "A Note on the Spatial Jet Instability of the Compressible Cylindrical Vortex Sheet," Deutsche Luft-und Raumfahrt Forschungsbericht, Berlin, DLR-FB 70-51, Oct. 1970.

[19]Strange, P.J.R. and Crighton, D.G., "Spinning Modes in Axisymmetric Jets," *Journal of Fluid Mechanics,* Vol. 134, Sept. 1983, pp. 231–245.

[20]Monkewitz, P.A. and Sohn, K.D., "Absolute Instability in Hot Jets and Their Control," AIAA Paper 86-1882, July 1986.

[21]Pierrehumbert, R.T., "Local and Global Baroclinic Instability of Zonally Varying Flow," *Journal of Atmospheric Sciences,* Vol. 41, No. 14, July 1984, pp. 2141–2162.

[22]Koch, W., "Local Instability Characteristics and Frequency Determination of Self-Excited Wake Flows," *Journal of Sound and Vibration,* Vol. 99, Jan. 1985, pp. 53–83.

[23]Koch, W., "Direct Resonances in Orr-Sommerfeld Problems," *Acta Mechanica,* Vol. 59, Pts. 1–2, May 1986, pp. 11–29.

[24]Monkewitz, P.A. and Nguyen, L.N., "Absolute Instability in the Near-Wake of Two-Dimensional Bluff Bodies," *Journal of Fluids and Structures,* Vol. 1, No. 2, April 1987, pp. 165–184.

[25]Chomaz, J.M., Huerre, P., and Redekopp, L.G., "Bifurcations to Local and Global Modes in Spatially Developing Flows," *Physical Review Letters,* Vol. 60, Jan. 1988, pp. 25–28.

[26]Monkewitz, P.A., "The Absolute and Convective Nature of Instability in Two-Dimensional Wakes at Low Reynolds Numbers," *Physics of Fluids,* Vol. 31, May 1988, pp. 999–1006.

[27]Strykowski, P., "The Control of Absolutely and Convectively Unstable Shear Flows," PhD. Thesis, Yale University, New Haven, CT, 1986.

[28]Papamoschou, D. and Roshko, A., "Observations of Supersonic Free Shear Layers," AIAA Paper 86-0162, Jan. 1986.

[29]Monkewitz, P.A. and Huerre, P., "Influence of the Velocity Ratio on the Spatial Instability of Mixing Layers," *Physics of Fluids,* Vol. 25, July 1982, pp. 1137–1143.

[30]Ahuja, K.K., Lepicovsky, J., and Brown, W.H., "Some Unresolved Questions on Hot-Jet Mixing Control through Artifical Excitation," AIAA Paper 86-1956, July 1986.